©2005. **I. V. Burskyy**

POINTWISE ESTIMATES OF A SOLUTION OF SOME BOUNDARY VALUE PROBLEM FOR THE EMDEN-FOWLER EQUATION

In this paper we propose some approaches for finding of pointwise estimates of a solution of the Dirichlet boundary value problem $-\Delta u \pm |u|^{q-1}u = 0$, $|u| = k$ when $|x| = d < 1$ and $|u| = 0$ when $|x| = 1$ where $x \in \Omega = \{x| \ d < |x| < 1\}$. Along with these we consider the same boundary conditions for the Laplace equation and get appropriate estimates for this easier case. We indicate some way that permits to find upper and lower estimates of a solution with explicit constants in turns.

1. Introduction.

In this paper we will consider the Dirichlet problem for the well-known Emden-Fowler equation. It will obtain several pointwise estimates of a solution of the problem:

$$-\Delta u + |u|^{q-1}u = 0, \quad q > 1, \quad x \in \Omega = B_1 \backslash B_d \tag{1.1}$$

$$u = 0 \qquad\qquad |x| = 1 \tag{1.2}$$

$$u = k \qquad\qquad |x| = d \tag{1.3}$$

for the case $n > 2$, $1 < q < \frac{n+2}{n-2}$.

The similar equation will be considered

$$\Delta u + |u|^{q-1}u = 0, \quad q > 1, \quad x \in \Omega = B_1 \backslash B_d \tag{1.4}$$

and the Laplace equation for a comparison

$$\Delta u = 0 \tag{1.5}$$

with the same boundary value conditions (1.2) and (1.3).

The second part of the paper is devoted to some general statements on existence and uniqueness of the solution. In the third part we obtain estimates of a solution that follow from the definition of a generalization solution and classical inequalities. By means of the same arguments there was obtained a comparison of estimate for solution of the problem (1.1), (1.2), (1.3) with estimate for solution of the problem (1.5), (1.2), (1.3). The main theorem of this part is:

Theorem 1. *For $n > 2$, $1 < q < \frac{n+2}{n-2}$*

1) the solution of the problem (1.1), (1.2), (1.3) satisfies the estimate:

$$u(x)\left(c_1 k + c_2 k^{q-1} d^2\right)\left(\frac{d}{|x|}\right)^{n-2},$$

2) the solution of the problem (1.5), (1.2), (1.3) and positive spherically symmetric solution of the problem (1.4), (1.2), (1.3) satisfies the estimate:

$$u(x) \le ck\left(\frac{d}{|x|}\right)^{n-2}.$$

In the fourth part it is suggested a method that based on comparison theorems and permit to obtain both upper and lower estimates which have some explicit constants.

THEOREM 2. *For the same n and q*

1) the solution of the problem (1.1), (1.2), (1.3) *has the following estimates*

$$C_1x^{2-n} + C_2x^2 + C_3 \leq u(x) \leq k\frac{x^{2-n}-1}{d^{2-n}-1},$$

here C_1, C_2, C_3 are some explicit constants (see formulae (4.8), (4.9)) in the case of small x and d coefficient C_1 has the following principal term:

$$\frac{k - \frac{k^q}{n(n-2)}}{d^{2-n}-1}.$$

The left side may have only restricted application, when k is not a big constant.

2) Positive spherical solution of the problem (1.4), (1.2), (1.3) has the following estimate

$$u(x) \geq k\frac{x^{2-n}-1}{d^{2-n}-1}.$$

If besides this solution is bounded above by constant k then the estimate holds

$$C_1'x^{2-n} + C_2'x^2 + C_3' \geq u(x) \geq k\frac{x^{2-n}-1}{d^{2-n}-1}.$$

with constants C_1', C_2', C_3'. Here for small d the coefficient C_1' has the principal term:

$$\frac{k + \frac{k^q}{n(n-2)}}{d^{2-n}-1}.$$

The estimates that were obtained in theorems and those of similar to them have important significance for studying nonlinear elliptic boundary value problems in domain with finely granulated boundary (see [1]). Equations (1.1) and (1.4) have also important physical applications. They can be met in astrophysics in the form of Emden equation and in atomic physics in the form of the Fermi-Thomas equation. Remark that the question of nonuniqueness for equation (1.4) was studied by S.Pohozhaev in [4].

2. Some general remarks.

Further we will consider a wide enough class of solutions $C^2(\bar{\Omega})$, that is spherically symmetric if it won't be underlined otherwise.

Remark, at first, that there exists a generalized solution of problem (1.2), (1.3) for the equation (1.1), (1.4) and (1.5) u (see the equality (3.2) for the equation (1.1)) from the Sobolev space $W_0^{1,2}(\Omega)$ of homogeneous Dirichlet problem for the case of $1 < q < \frac{n+2}{n-2}$ as it flows out from theorem 4.1 in [1] (In this case for $n > 2$ we have $W_0^{1,2}(\Omega) \subset L_q(\Omega)$). For nonhomogeneous boundary data one may carry on the same arguments for a remainder $u - \chi$.

Remark, secondly, that any generalized solution of homogeneous Dirichlet problem for nonhomogenious equation is arbitrarily smooth function because it assumes a raising of the smoothness by virtue of the ellipticity. Indeed, if we carry over the term $u^{q-1}|u|$ into the right-part side of the equation and we will suppose it belongs to any space then the same u as the solution of equation from left-part side with known right-part side will a function from more smooth space.

Note, more, that our boundary value problem assumes only unique solution for the equation (1.1) or (1.5) by virtue of usual arguments with the maximum principle. Therefore each solution of the problems (1.1), (1.2), (1.3) or (1.5), (1.2), (1.3) is spherically symmetric solution which satisfies the equation

$$u'' + \frac{n-1}{r}u' = u^q.$$

because respective boundary value problem (see (4.1)) has unique solution.

Let us, at last, give a proof that our solution of (1.1) is necessarily a positive function. Indeed, let the function u be negative at some point. Then it has a minimum at some point x_0. At this point we have $\nabla u(x_0) = 0, \Delta u(x_0) > 0$ but the right-side part of the equation is negative. Therefore $u > 0$.

3. The obtaining of pointwise estimates of Laplace equation solution and Emden-Fouler equation solution by means of the Mozer's inequality.

Here we will prove the theorem 1. We consider the boundary value problem (1.2), (1.3) for the equation (1.1) and for convenience below we will compare calculations for equations (1.5). And then we will remark the case of the equation (1.4).

1). First, we will obtain some auxiliary estimates of expressions

$$\int\limits_\Omega \left|\frac{\partial u}{\partial x}\right|^2 dx \text{ and } \int\limits_\Omega \left(\left|\frac{\partial u}{\partial x}\right|^2 + u^q\right) dx.$$

For this goal let us write integral identities for the equations (1.5) and (1.1) respectively that we will write as equalities

$$\sum_{i=1}^n \int\limits_\Omega \frac{\partial u}{\partial x_i}\frac{\partial \phi}{\partial x_i}\, dx = 0, \tag{3.1}$$

$$\int\limits_\Omega \left\{\sum_{i=1}^n \frac{\partial u}{\partial x_i}\frac{\partial \phi}{\partial x_i} + u^{q-1}\phi\right\} dx = 0 \tag{3.2}$$

that hold for all $\phi \in W_0^{1,2}(\Omega)$ Here we denote by $\phi \in W_0^{1,2}(\Omega)$ the usual Sobolev space of functions in Ω, that are square integrable with their first derivatives and that equal to zero on the boundary.

Let us take the function ϕ as the following

$$\phi(x) = u(x) - k\omega\left(\tfrac{|x|}{d}\right), \qquad 2d < 1,$$

where $\omega(s) \in C_0^\infty(\mathbf{R}^+)$, $\mathbf{R}^+ = [0, +\infty)$ is a function with properties $\omega(s) = 1, s \in [0,1]$; $\omega(s) = 0, s \in [2, \infty)$; $0 < \omega(s) < 1, s \in (1,2)$.

Remark, the function $\phi(x)$ is vanished for $|x|=1, |x|=d$ and

$$\int_\Omega \left|\frac{\partial u}{\partial x}\right|^2 dx = k\int_\Omega \sum_i \frac{\partial u}{\partial x_i}\frac{\partial \omega\left(\frac{|x|}{d}\right)}{\partial x_i}dx.$$

It is obvious that the estimate

$$\left|\frac{\partial \omega\left(\frac{|x|}{d}\right)}{\partial x_i}\right| = \left|\omega'\left(\tfrac{|x|}{d}\right)\frac{x_i}{d|x|}\right| \le \frac{C_1}{d}$$

.

Let us use together with well-known inequality $ab \le \epsilon a^2 + \frac{b^2}{\epsilon}$:

$$k\int_\Omega \sum_i \frac{\partial u}{\partial x_i}\frac{\partial \omega\left(\frac{|x|}{d}\right)}{\partial x_i}dx \le kn\int_{d\le|x|\le 2d}\left|\frac{\partial u}{\partial x}\right|\frac{C_1}{d}\,dx \le$$

$$\le \epsilon\int_\Omega \left|\frac{\partial u}{\partial x}\right|^2 dx + \frac{1}{\epsilon}k^2n^2\frac{C_1^2}{d^2}\int_{d\le|x|\le 2d}dx.$$

From here we have (for the equation (3.1))

$$\int_\Omega \left|\frac{\partial u}{\partial x}\right|^2 dx \le C_2k^2d^{n-2} \tag{3.3}$$

Let us now obtain an estimate for $\int_\Omega (\left|\frac{\partial u}{\partial x}\right|^2 + u^q)\,dx$, where u is a solution of (3.2). To this end we substitute the same function $\phi(x) = u(x) - k\omega\left(\frac{|x|}{d}\right)$ into integral identity (3.2):

$$\int_\Omega \left(\left|\frac{\partial u}{\partial x}\right|^2 + u^q\right)dx = k\int_\Omega \sum_{i=1}^n \frac{\partial u}{\partial x_i}\frac{\partial \omega\left(\frac{|x|}{d}\right)}{\partial x_i}dx + k\int_\Omega u^{q-1}\omega\left(\tfrac{|x|}{d}\right)dx \le$$

$$\le \int_{d\le|x|\le 2d}\left|\frac{\partial u}{\partial x}\right| kn\frac{C_1}{d}\,dx + \int_\Omega u^{q-1}k\omega\left(\tfrac{|x|}{d}\right)dx.$$

We used here previously obtained estimate for $\left|\frac{\partial \omega\left(\frac{|x|}{d}\right)}{\partial x_i}\right|$.

Then for the first term we use the inequality $ab \le \frac{a^2}{2} + \frac{b^2}{2}$ and for the second one the same inequality but with ϵ:

$$\frac{\epsilon ab}{\epsilon} \le \frac{\epsilon^p a^p}{p} + \frac{b^r}{\epsilon^r r},$$

where we take $p = q/(q-1), r = q$ and the constant ϵ is chosen so that $\epsilon^p/p = 1/2$. We continue the inequality:

$$\int_{d\le|x|\le 2d}\left|\frac{\partial u}{\partial x}\right| kn\frac{C_1}{d}\,dx + \int_\Omega u^{q-1}k\omega\left(\tfrac{|x|}{d}\right)dx \le$$

$$\leq \frac{1}{2}\int\limits_{d\leq|x|\leq 2d}\left|\frac{\partial u}{\partial x}\right|^2 dx + \frac{k^2n^2}{2}\int\limits_{d\leq|x|\leq 2d}\frac{C_1^2}{d^2}\,dx + \frac{1}{2}\int\limits_{\Omega}u^q\,dx + C_2\int\limits_{d\leq|x|\leq 2d}k^q\omega^q\,dx.$$

hence we have finally

$$\int\limits_{\Omega}\left(\left|\frac{\partial u}{\partial x}\right|^2 + u^q\right)dx \leq C_1k^2d^{n-2} + C_2k^qd^n\,, \tag{3.4}$$

where C_1, C_2 are some new constants.

2). Now we get over some auxiliary estimate of expression $\int\limits_{\{u<\mu\}}\left|\frac{\partial u}{\partial x}\right|^2 dx$ where $\mu:\ 0<\mu<k$. Take as ϕ the function $\phi = \min\{u(x),\mu\} - \frac{\mu}{k}u(x)$ and substitute it at first into (3.1) and then into (3.2).

It is easy to see that the function $\phi(x)$ vanish for $|x|=1$, $|x|=d$ and

$$\frac{\partial}{\partial x}\min\{u(x),\mu\} = \begin{cases}\partial u/\partial x, & \text{if } \{u<\mu\},\\ 0, & \text{if } \{u>\mu\}\end{cases}$$

$$\int\limits_{\{u<\mu\}}\left|\frac{\partial u}{\partial x}\right|^2 dx = \frac{\mu}{k}\int\limits_{\Omega}\left|\frac{\partial u}{\partial x}\right|^2 dx.$$

By using (3.3) we obtain the necessary estimate (for (3.1))

$$\int\limits_{\{u<\mu\}}\left|\frac{\partial u}{\partial x}\right|^2 dx \leq C_2\mu k d^{n-2}. \tag{3.5}$$

On the same way we can obtain the following estimate by means of the substitution ϕ into (3.2) and then using (3.4):

$$\int\limits_{\{u<\mu\}}\left|\frac{\partial u}{\partial x}\right|^2 dx + \int\limits_{\{u<\mu\}}u^q\,dx + \mu\int\limits_{\{u<\mu\}}u^{q-1}\,dx =$$

$$= \frac{\mu}{k}\int\limits_{\Omega}\left(\left|\frac{\partial u}{\partial x}\right|^2 + u^q\right)dx \leq \frac{\mu}{k}(C_2k^2d^{n-2} + C_3k^qd^n)$$

Thus, we come to the estimate for the equation (3.2)

$$\int\limits_{\{u<\mu\}}\left|\frac{\partial u}{\partial x}\right|^2 dx + \int\limits_{\{u<\mu\}}u^q\,dx + \mu\int\limits_{\{u<\mu\}}u^{q-1}\,dx \leq \mu(C_2kd^{n-2} + C_3k^{q-1}d^n) \tag{3.6}$$

3). In order to obtain a final estimate of the maximum module of solution we will use Mozer's method ([1], [2], [3]). It is valid the inequality

$$\max_{\Gamma_\rho}|u(x)|^2 \leq \frac{C}{\rho^n}\int\limits_{\Gamma(\rho)}u^2(x)\,dx, \tag{3.7}$$

where Γ_ρ is a spherical layer between spheres of radiuses $\rho+\epsilon$ and $\rho-\epsilon$, $\tilde{\Gamma}(\rho)$ is a wider spherical layer which encloses the layer Γ_ρ. We will need the following inequality of a type by Fridrichs or Poincare

$$\int\limits_{B(\rho)} |u(x)|^p \le C\rho^p \int\limits_{B(R)} \left|\frac{\partial u}{\partial x}\right|^2 dx, \qquad \text{where } \rho < R \tag{3.8}$$

Let us get over the main part of the calculations

$$\int\limits_{\Gamma(\rho)} u^2(x)\,dx = \int\limits_{\Gamma(\rho)} [min\{u(x),\mu\}]^2 \le \int\limits_{B(\rho+\epsilon)} min^2\{u(x),\mu\}\,dx \le$$

$$\le C\rho^2 \int\limits_{\{u<\mu\}} \left|\frac{\partial u}{\partial x}\right|^2 dx, \qquad \mu = \max_{\Gamma_\rho} u(x). \tag{3.9}$$

From (3.7), (3.9) and (3.5) we have

$$\mu^2 \le C\frac{\rho^2}{\rho^n}\mu k d^{n-2}$$

$$\mu \le Ck\left(\frac{d}{\rho}\right)^{n-2}$$

$$\max_{|x|=\rho}|u(x)| \le Ck\left(\frac{d}{\rho}\right)^{n-2}.$$

Because ρ can be an arbitrary number between d and 1, for the Laplace equation (3.1) one may read the following pointwise estimate

$$u(x) \le max|u(x)| \le Ck\left(\frac{d}{|x|}\right)^{n-2}. \tag{3.10}$$

Now, if one do the same for the equation (3.2), i.e. if one uses (3.7), (3.9) and (3.6), then he obtains

$$\mu \le C_1k\left(\frac{d}{\rho}\right)^{n-2} + C_2k^{q-1}d^2\left(\frac{d}{\rho}\right)^{n-2}$$

and finally

$$u(x) \le \max|u(x)| \le C_1k\left(\frac{d}{|x|}\right)^{n-2} + C_2k^{q-1}d^2\left(\frac{d}{|x|}\right)^{n-2} \tag{3.11}$$

Consider at last the case of the equation (1.4). For this goal let us write integral identity for the equation (1.4)

$$\int\limits_{\Omega}\left\{\frac{\partial u}{\partial x_i}\frac{\partial \phi}{\partial x_i} - u^{q-1}\phi\right\}dx = 0,$$

that holds for all $\phi \in W_0^{1,2}(\Omega)$. Here we do the same steps 1),2) and 3) as in the case of the equation (1.5) without any problems because the term with $-u^q$ is nonpositive. Therefore we obtain the same estimate as for the equation (1.5).

The theorem 1 is proved.

4. Some way of obtaining of upper and lower estimates of a solution of the Dirichlet boundary value problem for the Emden-Fouler equation.

We will consider a spherically symmetric solution of the problem (1.1), (1.2), (1.3) or problem (1.4), (1.2), (1.3) belonging to the space $C^2(\bar{\Omega})$ under condition $1 < q < (n+2)/(n-2)$. (see section 2). Firstly, let remark that if $U(x) = u(|x|)$ is the existing smooth solution of the problem (1.1), (1.2), (1.3) then the function $u(r)$ satisfies the equation

$$u'' + \frac{n-1}{r}u' = |u|^{q-1}u.$$

The proof flows out from an usual transfer to spherical variables. By means of solution positiveness our problem can be written as

$$\begin{cases} u'' + \frac{n-1}{r}u' = u^q \\ u(1) = 0, \quad u(d) = k. \end{cases} \tag{4.1}$$

Now let us formulate the main auxiliary result of this point.

LEMMA 1. *Let the functions $\hat{u}_1$, $\hat{u}_2$ be positive functions and the functions u_1, u_2 be solutions of the following problems*

$$\begin{cases} u_1'' + \frac{n-1}{r}u_1' = \hat{u}_1^q, \\ \\ u_1(1) = 0, \quad u_1(d) = k, \end{cases} \tag{4.2}$$

$$\begin{cases} u_2'' + \frac{n-1}{r}u_2' = \hat{u}_2^q, \\ \\ u_2(1) = 0, \quad u_2(d) = k. \end{cases} \tag{4.3}$$

Then $\hat{u}_1 \le \hat{u}_2$ implies $u_2 \le u_1$.

Proof. Let us consider the remainder of equations (4.3) − (4.2). After subtraction we will have

$$(u_2 - u_1)'' + \frac{n-1}{r}(u_2 - u_1)' = \hat{u}_2^q - \hat{u}_1^q \ge 0,$$

$$u_2(1) - u_1(1) = 0, \quad u_2(d) - u_1(d) = 0.$$

Let $u = u_2 - u_1 \ge 0$ at some point r. Then it has a maximum at some point r_0. At this point we have $u'(r_0) = 0, u''(r_0) < 0$ but the right-side part of the equation is positive. Therefore $u \le 0$ that is $u_2 \le u_1$.

Let us pass to description of the above-mentioned method.

COROLLARY 1. *Let u be a solution of the problem (4.1) and u_0 be any function that $u_0 \le u$. Let u_1 be a solution of the problem*

$$\begin{cases} u_1'' + \frac{n-1}{r}u_1' = u_0^q \\ u_1(1) = 0, \quad u_1(d) = k. \end{cases} \tag{4.4}$$

Then $u \le u_1$.

For proof we apply the lemma to the problems (4.1) and (4.4).

Corollary 2. *Let u be a solution of the problem (4.1) and u_1 be any function that $u_1 \geq u$. Let u_2 be a solution of the problem*

$$\begin{cases} u_2'' + \frac{n-1}{r} u_2' = u_1^q \\ u_2(1) = 0, \quad u_2(d) = k. \end{cases} \tag{4.5}$$

Then $u \geq u_2$.

For proof we apply the lemma 1 to the problems (4.1) and (4.5).

Let us consider our second problem with equation (1.4) that can be written as

$$\begin{cases} u'' + \frac{n-1}{r} u' = -u^q \\ u(1) = 0, \quad u(d) = k. \end{cases} \tag{4.5$'$}$$

Lemma 2. *Let the functions $\hat{u}_1$, $\hat{u}_2$ be positive functions and the functions u_1, u_2 be solutions of the following problems*

$$\begin{cases} u_1'' + \frac{n-1}{r} u_1' = -\hat{u}_1^q, \\ u_1(1) = 0, \quad u_1(d) = k \end{cases}$$

$$\begin{cases} u_2'' + \frac{n-1}{r} u_2' = -\hat{u}_2^q, \\ u_2(1) = 0, \quad u_2(d) = k \end{cases}$$

Then $\hat{u}_1 \leq \hat{u}_2$ implies $u_1 \leq u_2$.

The proof is analogous to the proof of the lemma 1.

Corollary 3. *Let u be a solution of the problem (4.5$'$) and u_0 be any function which $u_0 \leq u$. Let u_1 be a solution of the problem*

$$\begin{cases} u_1'' + \frac{n-1}{r} u_1' = -u_0^q, \\ u_1(1) = 0, \quad u_1(d) = k. \end{cases}$$

Then $u_1 \leq u$.

Corollary 4. *Let u be a solution of the problem (4.5$'$) and u_1 be any function which $u_1 \geq u$. Let u_2 be a solution of the problem*

$$\begin{cases} u_2'' + \frac{n-1}{r} u_2' = -u_1^q \\ u_2(1) = 0, \quad u_2(d) = k. \end{cases}$$

Then $u_2 \geq u$.

Thus, we can obtain a set of estimates of the solution u, lower and upper.

For further applications we do some calculations. Consider the problem

$$\begin{cases} w'' + \frac{n-1}{x} w' = f(x) \\ w(1) = 0, \quad w(d) = k. \end{cases}$$

If denote $f(x) = u^q, w = u$ then equality (4.1) can be written as such equation. Solving such ordinary linear differential equation and boundary value problem as usually, we receive

$$w(x) = \int_x^1 t^{1-n} \int_t^1 \tau^{n-1} f(\tau)\, d\tau dt + (k - D)\frac{x^{2-n} - 1}{d^{2-n} - 1}, \tag{4.6}$$

where $D = \int_d^1 t^{1-n} \int_t^1 \tau^{n-1} f(\tau)\, d\tau dt.$

Bring some calculations based on this approach.

I 1). Let $u_0 = 0$ for the equation (1.1). Because $u_0 \le u$ we have $u \le u_1$ from the corollary 1, that

$$u(x) \le k\frac{x^{2-n} - 1}{d^{2-n} - 1} \tag{4.7}$$

I 2) Now for the equation (1.1) we would like to take $u_1 = k$ because $u \le k$. We will receive

$$u_2 = \int_x^1 t^{1-n} \int_t^1 \tau^{n-1} k^q\, d\tau dt + (k - D_0)\frac{x^{2-n} - 1}{d^{2-n} - 1},$$

$$D_0 = \int_d^1 t^{1-n} \int_t^1 \tau^{n-1} k^q\, d\tau dt = \frac{k^q}{n}\left[\frac{1}{2-n} - \frac{d^{2-n}}{2-n} - \frac{1}{2} + \frac{d^2}{2}\right] =$$

$$= \frac{k^q}{2n(n-2)}(2d^{2-n} + (n-2)d^2 - n).$$

Denote

$$\tilde{C}_1 = \frac{k^q}{2n(n-2)}, \quad \tilde{C}_2 = -\frac{k - D_0}{d^{2-n} - 1}, \tag{4.8}$$

then

$$u_2 = \tilde{C}_1(2x^{2-n} + (n-2)x^2 - n) + \tilde{C}_2(1 - x^{2-n}) =$$
$$(2\tilde{C}_1 - \tilde{C}_2)x^{2-n} + \tilde{C}_1(n-2)x^2 + (\tilde{C}_2 - n\tilde{C}_1) = C_1 x^{2-n} + C_2 x^2 + C_3 \tag{4.9}$$

and corollary 2 says

$$u \ge C_1 x^{2-n} + C_2 x^2 + C_3.$$

Here for small d the coefficient $C_1 = 2\tilde{C}_1 - \tilde{C}_2$ has the following principal term:

$$\frac{k - \frac{k^q}{n(n-2)}}{d^{2-n} - 1},$$

therefore this formula can have only bounded application, i.e. when k is not a big constant.

II 1) Let $u_0 = 0$ for equation (1.4). Because it is considered a positive solution i.e. $u_0 \le u$ we have $u_1 \le u$ from the corollary 3 or

$$u(x) \ge k\frac{x^{2-n} - 1}{d^{2-n} - 1}$$

II 2) Now for the equation (1.4) we assume that $u \leq k$ and take $u_1 = k$. We will receive the same formula as in the point I 2) but with the change of k^q on $-k^q$. Then corollary 4 says

$$u \leq C_1' x^{2-n} + C_2' x^2 + C_3',$$

and for small d the coefficient C_1' has the principal term:

$$\frac{k + \frac{k^q}{n(n-2)}}{d^{2-n} - 1}.$$

The proof of the theorem 2 is over.

1. *Skrypnik I. V* Methods of investigation of nonlinear ellitic boundary value problems // Moscow.- Nauka.- 1990.
2. *Gilbarg D., TrudingerN.* Elliptic partial differential equations of second order // Springer-Verlag.- Berlin-Heidelberg-New York- Tokyo.- 1983.
3. *Mozer J.* A new proof of de Giorgi's theorem concerning the regularity problem for elliptic differential equations // Comm. Pure and Appl. Math.- 1960.- v. 13.- n.3.- p. 457-468.
4. *Pohozhaev S.I.* On eigenfunctions of the equation $\Delta u + \lambda f(u) = 0$ // Doklady Akademii Nauk SSSR.- 1965.- v. 165.- n. 1.- p. 36-39.

Department of Mathematics,
Donetsk National University,
Universitetskaja str., 24,
Donetsk, 83055, Ukraine
v30@dn.farlep.net

Received 1.09.2004